ZELEVINSKY INVOLUTION AND MOEGLIN-WALDSPURGER ALGORITHM FOR $\mathrm{GL}_n(D)$

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Abstract. In this short note, we remark that the algorithm of Moeglin and Waldspurger for computing the dual (as defined by Zelevinsky) of an irreducible representation of GL_n still works for the inner forms of GL_n, the proof being basically the same.

1. Segments, multisegments and the involution $^{\#}$

A **multiset** is a finite set with finite repetitions $(a, a, b, c, d, d, d, e, a, ...)$. A **segment** Δ is the void set or a set of consecutive integers $\{b, b+1, ..., e\}$, $b, e \in \mathbb{Z}$, $b \leq e$. We call e the **ending** of Δ and the integer $e - b + 1$ the **length** of Δ. By convention, the length of the void segment is 0. Let $\Delta = \{b, b+1, ..., e\}$ and $\Delta' = \{b', b'+1, ..., e'\}$ be two segments. We say Δ **precede** Δ' if $b < b'$, $e < e'$ and $b' \leq e + 1$. We also write $\Delta \geq \Delta'$ if $b > b'$ or $b = b'$ and $e \geq e'$. This is a total order on the set of segments.

A **multisegment** is a multiset of segments. We identify multisegments obtained from each other by dropping or adding void segments. The **full extended length** of a multisegment is the sum of the lengths of all its elements and is 0 if the multisegment is void. The **support** of a multisegment m is the multiset of integers obtained by taking the union (with repetitions) of the segments in m. A multisegment $(\Delta_1, \Delta_2, ..., \Delta_k)$ is said to be ordered if $(\Delta_1 \geq \Delta_2 \geq ... \geq \Delta_t)$. The lexicographic order induces a total order on ordered multisegments : if $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ and $m' = (\Delta'_1, \Delta'_2, ..., \Delta'_{t'})$ are multisegments, then $m \geq m'$ if $\Delta_1 > \Delta'_1$, or $\Delta_1 = \Delta'_1$ and $\Delta_2 > \Delta'_2$, and so on, or $t \geq t'$ and $\Delta_i = \Delta'_i$ for all $i \in \{1, 2, ..., t'\}$.

If $\Delta = \{b, b+1, ..., e\}$ is a segment, we set $\Delta^- = \{b, b+1, ..., e-1\}$ with the convention that Δ^- is void if $b = e$.

Let m be a multisegment. We associate to m a multisegment $m^{\#}$ in the following way : let d be the biggest ending of a segment in m. Then chose a segment Δ_{i_0} in m containing d and maximal for this property. Then we define the integers $i_1, i_2, ..., i_r$ inductively : Δ_{i_s} is a segment of m preceding $\Delta_{i_{s-1}}$ with ending $d - s$, maximal with these properties, and r is such that there's no possibility to find such a i_{r+1}. Set $m^- = (\Delta'_1, \Delta'_2, ..., \Delta'_t)$, where

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$\Delta'_i = \Delta_i$ if $i \notin \{i_0, i_1, ..., i_r\}$, and $\Delta'_i = \Delta_i^-$ if $i \notin \{i_0, i_1, ..., i_r\}$. Then $\{d-r, d-r+1, ..., d\}$ is the first segment of $m^\#$. Starting from the beginning with m^- what we have done with m, we find the second segment of $m^\#$, and so on (so at the end we have that $m^\#$ is the multiset union of $\{d-s, d-s+1, ..., d\}$ and $(m^-)^\#$). This multisegment $m^\#$ is independent of the choices made for the construction. The map $m \mapsto m^\#$ is an involution of the set of non void multisegments. It preserves the support.

2. Representations of G_n

2.1. **Generalities.** Let F be an non-Archimedean local field of any characteristic with norm $|\ |_F$. For all $n \in \mathbb{N}^*$ let G_n be the group $GL_n(F)$, $\mathcal{A}_n$ be the set of equivalence classes of smooth finite length representations of G_n and $\mathcal{R}_n$ be the Grothendieck group of smooth finite length representations of G_n. As usual, we will slightly abuse notation by identifying representations and their equivalence classes, and sometimes, representations with their image in the Grothendieck group $\mathcal{R}_n$.

The set B_n of classes of smooth irreducible representations of G_n is a basis of $\mathcal{R}_n$. If $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$, then $\pi_1 \otimes \pi_2$ is a representation of $G_{n_1} \times G_{n_2}$. This group may be seen as the subgroup L of matrices diagonal by two blocks of size n_1 and n_2 of $G_{n_1+n_2}$. We set

$$\pi_1 \times \pi_2 = \mathrm{ind}_P^{G_{n_1+n_2}}(\pi_1 \otimes \pi_2)$$

where "ind" is the normalized parabolic induction functor and P is the parabolic subgroup of $G_{n_1+n_2}$ containing L and the group of upper triangular matrices. We generalize this notation in an obvious way to any finite number of elements $\pi_i \in B_{n_i}$, $i \in \{1, 2, ..., k\}$.

Let $\mathcal{C}_n$ be the set of cuspidal representations of G_n and $\mathcal{D}_n$ the set of essentially square integrable representations of G_n (we assume irreducibility in the definition of cuspidal and essentially square integrable representations).

If χ is a smooth character of G_n and $\pi \in \mathcal{A}_n$, then $\chi\pi$ will denote the tensor product representation $\chi \otimes \pi$. Let ν_n be the character $g \mapsto |det(g)|_F$ of G_n. We will drop the index n when no confusion may occur.

2.2. **Irreducible representations.** Let $k \in \mathbb{N}^*$ and n_i, $i \in \{1, 2, ..., k\}$ be positive integers. For each i let $\sigma_i \in \mathcal{D}_{n_i}$. The representations σ_i being essential square integrable, for all $i \in \{1, 2, ..., k\}$ there exists a unique real number a_i such that $\nu^{a_i}\sigma_i$ is unitary. If the σ_i are ordered such that the sequence a_i is increasing, then $S = \sigma_1 \times \sigma_2 \times ... \times \sigma_k$ is called a **standard representation** and has a unique irreducible quotient $\theta(S)$. The representation S doesn't depend on the order of the σ_i as long as the condition that the sequence a_i is increasing is fullfiled. So S and $\theta(S)$ depend only on the multiset $(\sigma_1, \sigma_2, ..., \sigma_k)$. We call this multiset the **esi-support** of S or of $\theta(S)$ ("esi" : essentially square integrable).

2.3. **Standard elements.** The image in $\mathcal{R}_n$ of a standard representation is called a **standard element** of $\mathcal{R}_n$. The set H_n of standard elements of

$\mathcal{R}_n$ is a basis of $\mathcal{R}_n$. The map $W_n : S \mapsto \theta(S)$ is a bijection from H_n to B_n (see [DKV]).

2.4. **The involution.** On $\mathcal{R}_n$, we consider the involution I_n from [Au], which transforms irreducible representations to irreducible representations up to a sign. The involution commutes with induction ([Au]), i.e. if $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$, then $I_{n_1+n_2}(\pi_1 \times \pi_2) = I_{n_1}(\pi_1) \times I_{n_2}(\pi_2)$. Forgetting signs, the involution in [Au] gives rise to a permutation $|I_n|$ of B_n (which is the involution defined in [Ze]). We will call $|I_n|(\pi)$ the **dual** of π. See [Au] and [Ze].

The algorithm of Moeglin and Waldspurger ([MW]) computes the esi-support of the dual of a smooth irreducible representation π from the esi-support of π.

2.5. **Essentially square integrable representations.** Following [Ze], if k is a positive integer such that $k|n$, if we set $p = n/k$ and chose $\rho \in \mathcal{C}_p$, then $\rho \times \nu\rho \times \nu^2\rho \times, ..., \times \nu^{k-1}\rho$ has a unique irreducible quotient $Z(k, \rho)$ which is an essentially square integrable representation of G_n. Any element σ of $\mathcal{D}_n$ is obtained in this way and σ determines k and ρ such that $\sigma = Z(k, \rho)$. If $\rho \in \mathcal{C}_p$ for some p, given a segment $\Delta = \{b, b+1, ..., e\}$, we set

$$< \Delta >_\rho = Z(\nu^b\rho, e-b+1) \in \mathcal{D}_{p(e-b+1)}.$$

2.6. **Rigid representations.** If $\rho \in \mathcal{C}_p$ for some p we call the set $\{\nu^k\rho\}_{k\in\mathbb{Z}}$ **the ρ-line**. If $\pi \in B_n$ we say π is **ρ-rigid** if the cuspidal support of π is included in the ρ-line (of course, it is the $\nu\rho$-line too). An irreducible representation is called **rigid** if it is ρ-rigid for some ρ. If $\pi_1 \in B_{n_1}$ and $\pi_2 \in B_{n_2}$ are such that the cuspidal supports of π_1 and π_2 are disjoint, then $\pi_1 \times \pi_2$ is irreducible. So any $\pi \in B_n$ is a product of rigid representations π_i. Then we know ([Ze]) that the esi-support of π is the reunion with multiplicities of the esi-supports of the π_i. As I_n commutes with induction, to compute the esi-support of duals of irreducible representations, we need only to compute the esi-support of duals of rigid representations.

2.7. **Multisegments and representations.** If $m = (\Delta_1, \Delta_2, ..., \Delta_k)$ is an ordered multisegment of full length q and $\rho \in \mathcal{C}_p$, then m and ρ define a standard element $\pi_\rho(m)$ of $\mathcal{R}_{pq}$, precisely

$$\pi_\rho(m) =< \Delta_1 >_\rho \times < \Delta_2 >_\rho \times, ..., \times < \Delta_k >_\rho \in H_{pq},$$

and an irreducible representation

$$< m >_\rho = W_n(\pi_\rho(m)) \in B_{pq}.$$

The map $m \mapsto < m >_\rho$ realizes a bijection between the set of multisegments of full length q and the set $B_{n,\rho}$ of ρ-rigid irreducible representations of G_{pq}.

2.8. **The algorithm for G_n.** The result of Moeglin and Waldspurger in [MW] is : the dual of $< m >_\rho$ is $< m^\# >_\rho$.

2.9. **The proof.** We recall here their argument:

Let (p, ρ) be a couple such that p is a positive integer and $\rho \in \mathcal{C}_p$. Fix a multiset s with integer entries, and let S be the (finite) set of all the multisegments m having support s. They all have the same full length, let's call it k. Set $n = pk$. Let $B_\rho = \{< m >_\rho, m \in S\}$ and $H_\rho = \{\pi_\rho(m), m \in S\}$. Let $\mathcal{R}_\rho$ be the (finite dimensional) submodule of $\mathcal{R}_n$ generated by B_ρ. Then B_ρ and H_ρ are basis of the space $\mathcal{R}_\rho$. On B_ρ and H_ρ consider the decreasing order induced by the order on multisegments in S. Then we know that for this order the matrix M of H_ρ in the basis B_ρ is upper triangular and unipotent ([Ze] or [DKV]). The space $\mathcal{R}_\rho$ is stable under I_n. It is important to notice here that the involution $(-1)^{n-k} I_n$ of $\mathcal{R}_\rho$ transforms every irreducible representation in an irreducible one, since all the elements here have the same cuspidal support, of full length k (see [Au]). In other words, the restriction of $|I_n|$ to B_ρ is $(-1)^{n-k} I_n$.

Let T_1 (resp. T_2) be the matrix of the involution $(-1)^{n-k} I_n$ of $\mathcal{R}_\rho$ in the basis B_ρ (resp. H_ρ). *Then the matrix T_1 doesn't depend on the couple (p, ρ).* The argument, attributed in [MW] to Oesterlé, is the following:

We have already seen that T_1 is a permutation matrix ([Au]). Then as M is an upper triangular unipotent matrix, the relation $T_2 = M^{-1} T_1 M$ is a Bruhat decomposition for T_2 and this implies that T_1 is determined by T_2.

Now, T_2 itself doesn't depend on the couple (p, ρ) because:

(c1) if $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ with Δ_i of length n_i/p, then

$$I_n(\pi_\rho(m)) = I_{n_1}(< \Delta_1 >_\rho) \times I_{n_2}(< \Delta_2 >_\rho) \times ... \times I_{n_t}(< \Delta_t >_\rho),$$

(c2) if $\Delta = \{b, b+1, ..., e\}$, then $I_{(e+1-b)p}(< \Delta >_\rho) = (-1)^{(e+1-b)(p-1)} < m_\Delta >_\rho$, where $m_\Delta = (\{b\}, \{b+1\}, ..., \{e\})$,

(c3) one has $< m_\Delta >_\rho = \sum_{m' \leq m_\Delta} (-1)^{d(m')+e-b+1} \pi_\rho(m')$, where $d(m')$ is the cardinality of m' (as a multiset of segments) ([Ze]).

So it is enough to show that the dual of $< m >_\rho$ is $< m^\# >_\rho$ for a particular ρ. The authors conclude their proof by showing this relation holds for a clever choice of the cuspidal representation ρ.

3. Representations of G'_n

Let D be a central division algebra of dimension d^2 over F (with $d \in \mathbb{N}^*$) and let G'_n be the group $GL_n(D)$. We use the notation for objects relative to G_n, but with a prime, for objects relative to G'_n : $\mathcal{A}'_n$, $\mathcal{C}'_n$, $\mathcal{D}'_n$, $\mathcal{R}'_n$, B'_n... The involution I'_n ([Au]) on $\mathcal{R}'_n$, has the same properties as I_n : it transforms irreducible representations into irreducible representations, up to a sign, and commutes with induction.

If $g' \in G'_n$, one can define the characteristic polynomial $P_{g'} \in F[X]$ of g', and $P_{g'}$ is monic of degree nd ([Pi]). If $g' \in G'_n$, the determinant $det(g')$ of g' is the constant term of its characteristic polynomial. We write ν'_n for the character $g' \mapsto |det(g')|_F$ of G'_n, and we drop the index n when no confusion may occur.

For a given n, if $g \in G_{nd}$ and $g' \in G'_n$ we write $g \leftrightarrow g'$ if the characteristic polynomial of g is separable (i.e. has distinct roots in an algebraic closure of F) and is equal to the characteristic polynomial of g'. If $\pi \in \mathcal{R}_{nd}$ or $\pi \in \mathcal{R}'_n$, we denote by χ_π the character of π. It is well defined on the set of elements with separable characteristic polynomial even if the characteristic of F is not zero. The Jacquet-Langlands correspondence is the following result :

Theorem 3.1. *There exists a unique bijection* $\mathbf{C} : \mathcal{D}_{nd} \to \mathcal{D}'_n$ *such that for all* $\pi \in \mathcal{D}_{nd}$ *one has*

$$\chi_\pi(g) = (-1)^{nd-n}\chi_{\mathbf{C}(\pi)}(g')$$

for all $g \leftrightarrow g'$.

This well known result of [DKV] is also true in non-zero characteristic ([Ba1]).

One can extend the Jacquet-Langlands correspondence to a linear map between Grothendieck groups ([Ba2]) :

Proposition 3.2. *a) There exists a unique group morphism* $\mathbf{LJ} : \mathcal{R}_{nd} \to \mathcal{R}'_n$ *such that for all* $\pi \in \mathcal{R}_{nd}$ *one has*

$$\chi_\pi(g) = (-1)^{nd-n}\chi_{\mathbf{LJ}(\pi)}(g')$$

for all $g \leftrightarrow g'$.

The morphism $\mathbf{LJ}$ *is defined on the basis* H_{nd} *: if* $S = \sigma_1 \times \sigma_2 \times ... \times \sigma_k$, *with* $\sigma_i \in \mathcal{D}_{n_i}$, *then*

- if for all $i \in \{1, 2, ..., k\}$, $d|n_i$,

$$\mathbf{LJ}(S) = \mathbf{C}(\sigma_1) \times \mathbf{C}(\sigma_2) \times ... \times \mathbf{C}(\sigma_k),$$

- if not, $\mathbf{LJ}(S) = 0$.

b) For all $\pi \in \mathcal{R}_{nd}$, $\mathbf{LJ}(I_{nd}(\pi)) = (-1)^{nd-n} I'_n(\mathbf{LJ}(\pi))$.

The classification of irreducible representations is similar to the one for G_n, and we can define the esi-support of an irreducible representation, the standard elements H'_n and the bijection $W'_n : H'_n \to B'_n$. Knowing the esi-support of $\pi' \in B'_n$, one would like to compute the esi-support of $|I'_n|(\pi')$.

The classification of essentially square integrable representations on G'_n differs slightly from that on G_n (it is more general, since $G'_n = G_n$ when $D = F$). If $\rho' \in \mathcal{D}'_n$, then $\mathbf{C}^{-1}(\rho') \in \mathcal{D}_{nd}$. Following [Ta], if $\mathbf{C}^{-1}(\rho') = Z(k, \rho)$, we set $s(\rho') = k$, and $\nu_{\rho'} = (\nu')^{s(\rho')}$. Given a positive integer k such that $k|n$ and a $\rho' \in \mathcal{C}'_p$ where $p = n/k$, the representation $\rho' \times \nu_{\rho'}\rho \times \nu^2_{\rho'}\rho' \times ... \times \nu^{k-1}_{\rho'}\rho'$ has a unique irreducible quotient σ' which is an essentially square integrable representation of G'_n. We set then $\sigma' = T(k, \rho')$. Any $\sigma' \in \mathcal{D}'_n$ is obtained in this way and σ' determines k and ρ' such that $\sigma' = T(k, \rho')$. See [Ta] for details.

If $\rho' \in \mathcal{C}_p$ for some p, given a segment $\Delta = \{b, b+1, ..., e\}$, we set

$$< \Delta >_{\rho'} = T(\nu^b_{\rho'}\rho', e - b + 1) \in \mathcal{D}'_{p(e-b+1)}.$$

A line in this setting is a set of the form $\{\nu^k_{\rho'}\rho'\}_{k \in \mathbb{Z}}$ where ρ' is a cuspidal representation. The definition of ρ'-rigid and rigid representations and their properties are similar to the ones for G_n, and as for G_n, one needs only to compute of the esi-support of the duals for rigid representations.

If $m = (\Delta_1, \Delta_2, ..., \Delta_k)$ is an ordered multisegment and $\rho' \in \mathcal{C}'_p$, then m and ρ' define a standard element of some $\mathcal{R}'_n$, more precisely

$$\pi'_{\rho'}(m) =< \Delta_1 >_{\rho'} \times < \Delta_2 >_{\rho'} \times, ..., \times < \Delta_k >_{\rho'},$$

and an irreducible representation

$$< m >_{\rho'} = W'_n(\pi'_{\rho'}(m)).$$

The map $\pi'_{\rho'}$ realizes a bijection between the set of multisegments of full length k and the set of ρ'-rigid representations of G'_{pk}. Now, we claim that the algorithm for G'_n is the same as for G_n, namely :

Theorem 3.3. *The dual of the representation $< m >_{\rho'}$ is $< m^{\#} >_{\rho'}$.*

For the proof, we follow the argument in [MW] :

Let (p, ρ') be a couple such that p is a positive integer and $\rho \in \mathcal{C}'_p$, let k be a positive integer and set $n = pk$. Let $B'_{\rho'} = \{< m >_{\rho'}, m \in S\}$ and $H'_{\rho'} = \{\pi'_{\rho'}(m), m \in S\}$ (S has already been defined in the section 2.9). Let $\mathcal{R}'_{\rho'}$ be the finite dimensional submodule of $\mathcal{R}'_n$ generated by $B'_{\rho'}$. Then $B'_{\rho'}$ and $H'_{\rho'}$ are bases of $\mathcal{R}'_{\rho'}$. On $B'_{\rho'}$ and $H'_{\rho'}$ consider the decreasing order induced by the order on multisegments in S. Then the matrix M' of $H'_{\rho'}$ in the basis $B'_{\rho'}$ is upper triangular and unipotent ([DKV] and [Ta]). The involution $(-1)^{n-k} I'_n$ induces an involution of $\mathcal{R}'_{\rho'}$ which carries irreducible representations to irreducible representations. Let T'_1 (resp. T'_2) be the matrix of this involution in the basis $B'_{\rho'}$ (resp. $H'_{\rho'}$).

As for G_n, the matrix T'_1 doesn't depend on (p, ρ'), because Oesterlé's argument works again. First of all (see [Au]), T'_1 is a permutation matrix so the relation $T'_2 = M'^{-1} T'_1 M'$ is a Bruhat decomposition for T'_2 and this implies that T'_1 is determined by T'_2.

As for G_n, T'_2 itself doesn't depend on (p, ρ') because, as we will explain shortly afterwards, we have :

(c'1) If $m = (\Delta_1, \Delta_2, ..., \Delta_t)$ with Δ_i of length n_i/p, then

$$I'_n(\pi'_{\rho'}(m)) = I'_{n_1}(< \Delta_1 >_{\rho'}) \times I'_{n_2}(< \Delta_2 >_{\rho'}) \times ... \times I'_{n_t}(< \Delta_t >_{\rho'}).$$

(c'2) If $\Delta = \{b, b+1, ..., e\}$, then $I'_{(e+1-b)p}(< \Delta >_{\rho'}) = (-1)^{(e+1-b)(p-1)} < m_\Delta >_{\rho'}$, where $m_\Delta = (\{b\}, \{b+1\}, ..., \{e\})$.

(c'3) One has $< m_\Delta >_{\rho'} = \sum_{m' \leq m_\Delta} (-1)^{d(m')+e-b+1} \pi'_{\rho'}(m')$, where $d(m')$ is the cardinality of m' (as a multiset of segments).

The relation (c'1) is clear since the involution commutes with induction ([Au]).

(c'2) is true too: from the formula for I'_n to be found in [Au], and the computation in [DKV] of all normalized parabolic restrictions of essentially square integrable representations of G'_n, one may see $I'_{(e+1-b)p}(< \Delta >_{\rho'})$ is an alternate sum of representations $\pi'_{\rho'}(m_i)$, where m_i runs over the set of multisegments with same support as Δ. It is obvious that the maximal one is $\pi'_{\rho'}(m_\Delta)$. It appears in the sum with coefficient $(-1)^{(e+1-b)(p-1)}$, and so $W'_n(\pi'_{\rho'}(m_\Delta)) =< m_\Delta >_{\rho'}$, has to appear with coefficient $(-1)^{(e+1-b)(p-1)}$ in the final result. As we know a priori that this result is plus or minus an irreducible representation, (c'2) follows.

(c'3) is the combinatorial inversion formula ([Ze]), which is still true here since for all $m' \leq m_\Delta$ one has $\pi'_{\rho'}(m') = \sum_{m'' \leq m'} < m'' >_{\rho'}$.

So it is enough to show that the dual of $< m >_{\rho'}$ is $< m^{\#} >_{\rho'}$ for a particular ρ'. Or, equivalently, to show that for some ρ' we have $T'_2 = T_2$. Let $\rho \in \mathcal{C}_d$ and set $\rho' = \mathbf{C}(\rho)$. Then $\rho' \in \mathcal{C}'_1$ and $s(\rho') = 1$. The map **LJ** induces a bijection from H_ρ to $H'_{\rho'}$ commuting with the bijections from S onto these sets. The point b) of the proposition 3.2 implies then $T'_2 = T_2$.

4. Bibliography

[Au] A.-M. Aubert, Dualité dans le groupe de Grothendieck de la catégorie des représentations lisses de longeur finie d'un groupe réductif p-adique, *Trans. Amer. Math. Soc.*, 347 (1995), 2179-2189 (there's an Erratum: *Trans. Amer. Math. Soc.*, 348 (1996), 4687-4690).

[Ba1] A.I. Badulescu, Correspondance de Jacquet-Langlands en caractéristique non nulle, *Ann. Scient. Éc. Norm. Sup.* 35 (2002), 695-747.

[Ba2] A.I. Badulescu, Jacquet-Langlands et unitarisabilité, to appear in *Journal de l'Institut de Mathématiques de Jussieu*, may be found at http://www-math.univ-poitiers.fr/~ badulesc/articole.html.

[DKV] P. Deligne, D. Kazhdan, M.-F. Vignéras, Reprsentations des algbres centrales simples p-adiques, *Reprsentations des groupes rductifs sur un corps local*, Hermann, Paris 1984.

[MW] C. Moeglin, J.-L. Waldspurger, Sur l'involution de Zelevinski, *J. Reine Angew. Math.* 372 (1986), 136–177.

[Pi] R.S. Pierce, *Associative algebras*, Grad. Texts in Math., Vol 88, Springer-Verlag.

[Ta] M. Tadić, Induced representations of $GL(n; A)$ for a p-adic division algebra A, *J. Reine angew. Math.* 405 (1990), 48-77.

[Ze] A. Zelevinsky, Induced representations of reductive p-adic groups II, *Ann. Scient. Éc. Norm. Sup.* 13 (1980), 165-210.

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