Generalization of flat bundles with harmonic metrics (C.S.)

- Category of ***polarizable regular twistor $\mathcal{D}$-modules*** of weight w.

Process analogous to that of M. Saito, constructing a category of ***polarizable Hodge $\mathcal{D}$-modules***.

***Theorem* (C.S.)**

(1) *Kashiwara's conjecture (decomposition theorem) in this category.*

(2) *Smooth polarizable twistor $\mathcal{D}$-modules of weight $\mathbf{0}$ correspond to harmonic flat bundles.*

Comparison results

On a punctured Riemann surface (mainly follows from Simpson 1990):

(C.S.) Polarized regular twistor $\mathcal{D}$-modules of weight $\mathbf{0}$ on a ***compact*** Riemann surface X correspond to ***tame harmonic flat bundles*** on $X^* = X \setminus$ singular points.

(T.M.) Polarized regular twistor $\mathcal{D}$-modules of weight $\mathbf{0}$ on an open disc X with singularity at the center correspond to ***harmonic flat bundles*** on X^* which are tame at the center of the disc.

On the complement of a normal crossing divisor (Mochizuki 03)

Let X be a product of open discs and let D be a union of coordinate hyperplanes, $X^* = X \setminus D$.

Theorem* (Mochizuki)**. *Polarized regular twistor* $\mathcal{D}$*-modules of weight* $\mathbf{0}$ *on* X *with singularities along* D *correspond to* ***harmonic flat bundles *on* X^* *which are tame along* D.

Global theory

Theorem. *Let X be a projective complex manifold and let D be a normal crossing divisor in X. Set $X^* = X \smallsetminus D$.*

Semisimple representations of $\pi_1(X^*)$

$$\updownarrow$$

harmonic flat bundles on X^* which are tame along D.

Proof

(Corlette 88): The case $D = \emptyset$.

(Simpson 90): The case $\dim X = 1$.

(Mochizuki 04 using results of Jost-Zuo): The general case. $\square$

***Corollary* (C.S. (1) and Mochizuki 03)**. *Let Z be an irreducible projective variety over $\mathbb{C}$. We have then an equivalence:*

Semisimple representations of $\pi_1(Z^o)$
(Z^o *some smooth Zariski non empty open subset of* Z)

$$\updownarrow$$

Semisimple perverse sheaves strictly supported by Z
($= \mathrm{IC}_Z(\mathcal{L}),\ \mathcal{L}$ *semisimple local system on some* Z^o)

$$\updownarrow$$

Polarized regular twistor $\mathcal{D}$*-modules of weight* 0 *strictly supported by* Z
($=$ *tame harmonic flat bundle on some* Z^o)

Proof of the conjecture of Kashiwara

X, Y complex projective manifolds, $f : X \longrightarrow Y$ a morphism.