CHAPTER 14

POLARIZABLE HODGE MODULES OF NORMAL CROSSING TYPE

Summary.

14.1. Introduction

The local setting is as follows. The space X is a polydisc in $\mathbb{C}^n$ with analytic coordinates $x_1, \dots, x_n$, we fix $\ell \leqslant n$ and we denote by D the divisor $\{x_1 \cdots x_\ell = 0\}$. We also denote by D_i $(i \in I)$ the smooth components of D and by $D_{(\ell)}$ their intersection $D_1 \cap \cdots \cap D_\ell$. We will shorten the notation $\mathscr{O}_{D_{(\ell)}}[x_1, \dots, x_\ell]$ into $\mathscr{O}_{D_{(\ell)}}[x]$ and $\mathscr{D}_{D_{(\ell)}}[x_1, \dots, x_\ell]\langle \partial_{x_1}, \dots, \partial_{x_\ell}\rangle$ into $\mathscr{D}_{D_{(\ell)}}[x]\langle \partial_x\rangle$.

14.2. Synopsis

Given a pVHS on $(\Delta^*)^\ell \times \Delta^{n-\ell}$, consider Deligne's meromorphic extension $(\mathcal{V}_*, \nabla)$ and $\mathcal{V}_*^{>-\mathbf{1}} = \bigcap_{i=1}^\ell V_{(i)}^{>-1}\mathcal{V}_*$. Let $\mathcal{V}_{\min}$ be the corresponding minimal extension.

(1) Notice that $\mathcal{V}_*$ and $\mathcal{V}_{\min}$ have normal crossing type along D.

(2) To show that $F^p\mathcal{V}$ can be extended to $F^p\mathcal{V}_*^{>-\mathbf{1}}$ as a holomorphic bundle in such a way that $(F^\bullet\mathcal{V}_*^{>-\mathbf{1}}, (V_{(i)}^{>-1}\mathcal{V}_*^{>-\mathbf{1}})_{i\in I})$ are compatible filtrations of $\mathcal{V}_*^{>-\mathbf{1}}$.

(3) Extend this to $\mathcal{V}_{\min}$ by using Proposition 11.2.28.

(4) To show that the pairing Q on $\mathcal{V}$ extends as a pairing on $\mathcal{V}_*$ with values in moderate distributions. Show that its restriction to $\mathcal{V}_{\min}$ takes values in distributions, and its restriction to $\mathcal{V}_*^{>-\mathbf{1}}$ takes values in L^1_{loc}.

(5) To classify sesquilinear pairings on $\mathscr{D}$-modules of normal crossing types. They should be determined by their restriction to the $M_{\boldsymbol{\alpha}}$, $\boldsymbol{\alpha} \in [-1, 0]^\ell$.

(6) For $g(x) = x^{\boldsymbol{a}}$, compute $\psi_{g,\lambda}\mathrm{Q}$ by giving its restrictions to the $\widetilde{N}_{\boldsymbol{\beta}}$. In fact, one only needs to compute $\mathrm{P}_\ell\psi_{g,\lambda}\mathrm{Q}$.

(7) To be completed.

14.3. Comments

Here come the references to the existing work which has been the source of inspiration for this chapter.